

Kelvin-Helmholtz Instabilities at the Interface Between an Accretion Disk and the Magnetosphere of a Neutron Star

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To Professor Arnulf Schlüter on his 60th Birthday

The material in an accretion disk moves on Kepler orbits whereas the magnetosphere of a neutron star rotates like a solid body. Therefore large velocity differences will occur at the interface between the two media. These give rise to Kelvin-Helmholtz instabilities. We present a stability analysis for a simplified geometry. The consequences of the instability for the mass flow from the disk into the magnetosphere are discussed. The resulting transfer of angular momentum to the neutron star is compared with the observed changes of the rotation periods of these stars.

1. Introduction

Satellite observations have revealed the existence of regularly pulsating X-ray sources in our galaxy and in M31. 18 such sources have been discovered so far, with periods between 0.7 sec (SMC X-1) and 835 sec (X Per) [1]. On the average their periods decrease with time. The X-ray luminosities vary between 10^{34} erg/sec (X Per) and 6×10^{38} erg/sec (SMC X-1). ([2] gives a review of the physics of regularly pulsating X-ray sources.) Her X-1 with a period of 1.24 sec and a luminosity of 10^{37} erg/sec (i.e. 10^4 times the solar luminosity) is a typical specimen from this class of objects.

The generally accepted picture is that of a rotating neutron star with a strong magnetic field orbiting a normal optical companion. The energy source for the X-rays comes from gas which flows over from the companion star and accretes in the deep gravitational well of the neutron star. The rotation of the neutron star provides the clock-mechanism for the pulses. The observed pulses must be produced by an asymmetry in the infall of matter, and the obvious way to achieve this is to have inclined to the axis of rotation, a stellar magnetic field, which guides the matter down to the surface, such that only certain parts on the surface accrete. As these hot spots move through our field of vision, we see X-ray pulses with a period corresponding to the rotation period of the neutron star. This picture is strongly supported

by the measurement of a spectral line of Her X-1 [3] which can be interpreted as a cyclotron line implying a magnetic field of $B = 5 \times 10^{12}$ Gauss.

It is probable that because of the angular momentum of the accreting gas, the mass flow will lead to the formation of an extended flat disk around the neutron star. The accreting plasma is transported radially inwards through the disk and meets a stellar magnetic field of steadily increasing energy density. Because of the high electrical conductivity of the disk, we can assume that the neutron star's magnetic fields do not penetrate the disk. We then have a low density magnetosphere surrounding the high density disk. If we take the star's rotation axis perpendicular to the plane of the disk, and the star's magnetic field as a dipole in the plane of the disk, we arrive at a rather simple geometrical configuration (Figure 1). This is a very special configuration, but we have shown [4] that the angular momentum transfer by accretion, if it accelerates the star, leads to a secular motion of an arbitrary oblique dipole field towards the configuration shown in Figure 1.

The vertical structure of the accretion disk is given by hydrostatic equilibrium with a scale height

$$H^2 = r^2 (c_s/v_k)^2,$$

where c_s is the sound speed, and v_k the Keplerian velocity. The outer boundary in the z -direction is determined by pressure equilibrium of the gas pressure in the disk and the magnetic pressure $B^2/8\pi$. The inner edge of the disk at r_M is the point where $B^2/8\pi$ equals the gas pressure at the center of the disk. The boundary between disk and

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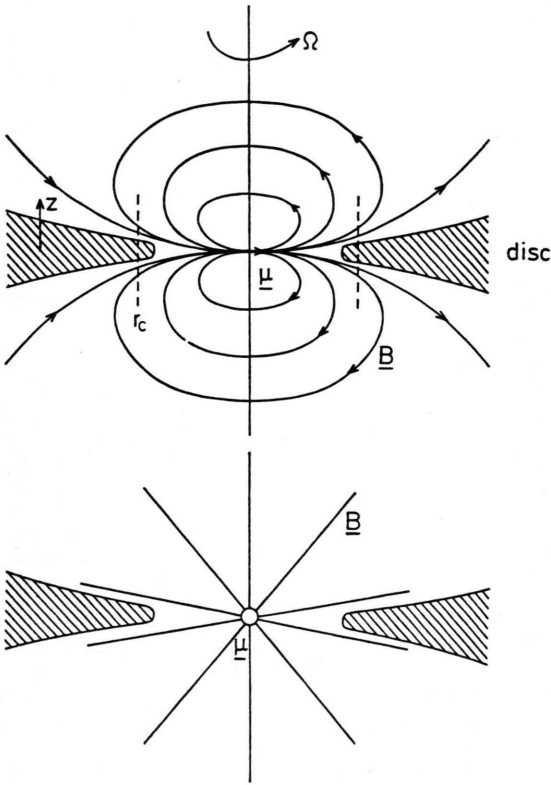


Fig. 1. Model of the magnetosphere-disk configuration. The upper part shows the side view of the magnetic dipole field, the lower part gives the projection of the field lines when the dipole is seen end-on.

magnetosphere is unstable, because of the velocity difference between the corotating magnetic field ($V_\phi = \Omega r$) and the Keplerian motion in the disk ($V_\phi = (GM/r)^{1/2}$). This difference velocity will give rise to a Kelvin-Helmholtz instability away from the radius of corotation $r_c = (GM/\Omega^2)^{1/3}$. In the following we want to discuss this instability in simple geometric configurations.

2. The Kelvin-Helmholtz Instability of a Discontinuous Interface

We take two homogeneous media separated by a planar interface. Medium 2 moves relative to medium 1 with a constant velocity Δv parallel to this interface (we shall use index 1 resp. 2 for the two media and take $z < 0$ for medium 1 and $z > 0$ for medium 2). We discuss the behaviour of small perturbations of the form

$$\xi = \xi_0 \exp \{i(kx - \omega t)\}. \quad (1)$$

Fejer [5] gives the dispersion relation for such a configuration.

$$\frac{\rho_1^2 [V_1 \cos^2(\alpha - \beta_1) - w^2]^2}{w^4} \quad (2)$$

$$1 - \frac{w^2(u_1^2 + V_1^2) - u_1^2 V_1^2 \cos^2(\alpha - \beta_1)}{\rho_2^2 [V_2^2 \cos^2(\alpha - \beta_2) - (w - \Delta v \cos \alpha)^2]^2} = \frac{(w - \Delta v \cos \alpha)^4}{1 - \frac{(w - \Delta v \cos \alpha)^2(u_2^2 + V_2^2) - u_2^2 V_2^2 \cos^2(\alpha - \beta_2)}{w^4}}.$$

The symbols have the following meaning: α angle between Δv and tangential wave vector k_t ($= (k_x, k_y, 0)$), β_i angle between Δv and B_i where B_i is the magnetic field which is parallel to the interface, $w = \omega/k_t$, ρ_i density, u_i sound velocity and V_i Alfvén velocity, $i = 1, 2$.

We simplify the configuration further by taking $V_1 = 0$ and $u_2 = 0$ (i.e. no magnetic field in medium 1 and dominating Alfvén velocity in medium 2). Setting $v_A = V_2$, $c_s = u_1$, $X = \Delta v \cos \alpha / v_A$ and $m = w/c_s$ one obtains from (2)

$$\rho_1^2 w^4 \left[1 - \left(m \frac{c_s}{v_A} - X \right)^2 \right] \quad (3)$$

$$= \rho_2^2 v_A^4 \left[\cos^2(\alpha - \beta) - \left(m \frac{c_s}{v_A} - X \right)^2 \right]^2 (1 - m^2).$$

Pressure equilibrium gives $\rho_1 c_s^2 = \frac{1}{2} \rho_2 v_A^2$ and we can therefore write

$$\frac{m^4}{4} \left[1 - \left(m \frac{c_s}{v_A} - X \right)^2 \right] \quad (4)$$

$$= \left[\cos^2(\alpha - \beta) - \left(m \frac{c_s}{v_A} - X \right)^2 \right]^2 (1 - m^2).$$

A sufficient condition for stability is obtained when all roots of (4) are real. Defining $Y = m/a - X$ with $a = v_A/c_s$ we can rewrite (4) as

$$(1 - Y^2)(X + Y)^4 \quad (5)$$

$$= 4(\cos^2(\alpha - \beta) - Y^2)^2 \left(\frac{1}{a^4} - \frac{(Y + X)^2}{a^2} \right).$$

By assuming $1/a \ll 1$ we then can expand the solutions of (5) in power series of $1/a$. There are various regimes for the solutions Y :

A) For Y of order unity there is one root $Y_1 \approx 1$, or $m_1 \approx a(1 + X)$, to lowest order of $1/a$; the next order leads to

$$Y_1 \approx 1 + \frac{2}{a^2} \frac{(\cos^2(\alpha - \beta) - 1)^2}{(1 + X)^2}. \quad (6)$$

Therefore up to this order m_1 is real.

B) For $|1 - X| \gg 1/a$ there is yet another solution of order unity: $Y_2 \approx -1$, or $m_2 \approx -a(1 - X)$ and in next order

$$Y_2 \approx -1 - \frac{2}{a^2} \frac{(\cos^2(\alpha - \beta) - 1)^2}{(1 - X)^2}, \quad (7)$$

which is also real.

C) Next we take $Y + X = \varepsilon \ll 1$; then (5) gives

$$\varepsilon^4 = \frac{4(\cos^2(\alpha - \beta) - (X - \varepsilon)^2)^2}{1 - (X - \varepsilon)^2} \left(\frac{1}{a^4} - \frac{\varepsilon^2}{a^2} \right) \quad (8)$$

and for $|1 - X| \gg \varepsilon$ one obtains

$$\varepsilon^4 = \frac{4(\cos^2(\alpha - \beta) - Y^2)^2}{1 - X^2} \left(\frac{1}{a^4} - \frac{\varepsilon^2}{a^2} \right). \quad (9)$$

The roots are given by

$$m^2 = a^2 \varepsilon^2 = \frac{2}{X^2 - 1} \cdot \{(\cos^2(\alpha - \beta) - X^2)^2 \pm [(\cos^2(\alpha - \beta) - X^2)^4 - (X^2 - 1) \cdot (\cos^2(\alpha - \beta) - X^2)^{1/2}]\} \quad (10)$$

and they provide the remaining solutions m_3, \dots, m_6 .

For stability one must have $m^2 > 0$ which requires $X^2 > 1$. For stability the expression under the square root of (10) must be positive. This is equivalent to

$$(X^2 - \cos^2(\alpha - \beta))^2 > X^2 - 1 \quad (11)$$

and gives an additional condition on X only if $\cos^2(\alpha - \beta) > \frac{3}{4}$. Then it leads to

$$X^2 > \cos^2(\alpha - \beta) + \frac{1}{2} \pm (\cos^2(\alpha - \beta) - \frac{3}{4})^{1/2}. \quad (12)$$

In Fig. 2 we have shaded the region of stability resulting from inequality (12) and $X^2 > 1$.

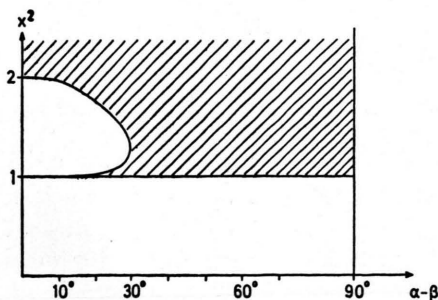


Fig. 2. Region of exponentially growing modes.

The results obtained so far are only valid if X differs sufficiently from unity. We shall discuss the solutions of (5) for the special case of $X = 1$. One finds that m_1 is not affected, but m_2, \dots, m_6 have to be evaluated differently. One obtains

$$\varepsilon^6 = 2(\cos^2(\alpha - \beta) - 1)^2 \left(\frac{1}{a^4} - \frac{\varepsilon^2}{a^2} \right) \quad (13)$$

with $\varepsilon = Y + 1$. This gives in lowest order in $1/a$

$$\varepsilon_{2,3} \approx \pm 1/a \quad (14)$$

and

$$\varepsilon_{4,5,6} \approx (-2(\cos^2(\alpha - \beta) - 1)^2)^{1/3} a^{-2/3} \cdot \exp\{i 2\pi n/3\}, \quad n = 0, 1, 2. \quad (15)$$

Using these results to compute higher order corrections leads to

$$\varepsilon_{2,3} \approx \frac{1}{a} \left(\pm 1 - \frac{1}{4(\cos^2(\alpha - \beta) - 1)^2} \frac{1}{a} \right) \quad (16)$$

and

$$\begin{aligned} \varepsilon_{4,5,6} \approx & -(2(\cos^2(\alpha - \beta) - 1)^2)^{1/3} a^{-2/3} \\ & \cdot \exp\{i 2\pi n/3\} \\ & - (2(\cos^2(\alpha - \beta) - 1)^2)^{1/3} \\ & \cdot (2 - 2\cos^2(\alpha - \beta)) \\ & - \frac{1}{3} \cos^4(\alpha - \beta) a^{-4/3} \\ & \cdot \exp\left\{-\frac{i 2\pi n}{3}\right\}. \end{aligned} \quad (17)$$

It can be seen from (15) that ε_5 gives exponentially growing modes with

$$\gamma \approx v_A^{1/3} c_s^{2/3} k_t \sqrt{3} (\frac{1}{2}(\cos^2(\alpha - \beta) - 1)^{2/3}). \quad (18)$$

We have found that for $X^2 < 1$ roots with $m^2 < 0$ occur, leading to exponentially growing modes. But only those modes which are concentrated around $z = 0$ are physically acceptable. This leads to the demand that in medium 1 $\text{Im}(k_{z1}) < 0$ and in medium 2 $\text{Im}(k_{z2}) > 0$ should hold. Fejer [5] gives the relation between k_z and m

$$(k_{z1}/k_t)^2 = -1 + m^2 \quad (19)$$

and

$$(k_{z2}/k_t)^2 = -1 + \left(\frac{m}{a} - X \right)^2 \quad (20)$$

and the condition for continuity at the interface

$$-\frac{m^2}{k_{z1}} = 2 \frac{\cos^2(\alpha - \beta) - \left(\frac{m}{a} - X \right)^2}{k_{z2}}. \quad (21)$$

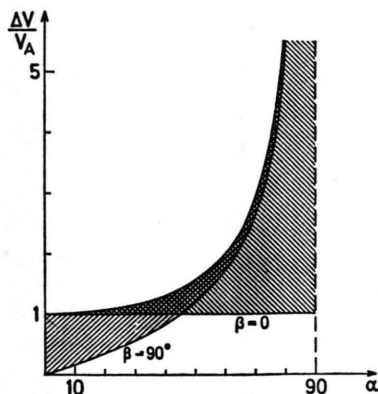


Fig. 3. Regions of instability for the cases $\beta = 0$ and $\beta = 90^\circ$.

Since one has $m^2 < 0$ (for $X^2 < 1$) and $m/a \approx 0$ one finds that k_{z1} and k_{z2} are imaginary in lowest order of $1/a$. From relation (21) and the requirement that k_{z1} and k_{z2} have opposite signs one thus deduces the condition

$$X^2 - \cos^2(\alpha - \beta) > 0. \quad (22)$$

One then arrives at the following limits on $\Delta v/v_A$ for the unstable modes

$$\frac{\cos^2(\alpha - \beta)}{\cos^2 \alpha} < \left(\frac{\Delta v}{v_A} \right)^2 < \frac{1}{\cos^2 \alpha}. \quad (23)$$

The special cases $\beta = 0$ ($\Delta v \parallel \mathbf{B}$) and $\beta = \pi/2$ ($\Delta v \perp \mathbf{B}$) give

$$1 < (\Delta v/v_A)^2 < 1/\cos^2 \alpha \quad \text{and}$$

$$\text{tg}^2 \alpha < (\Delta v/v_A)^2 < 1/\cos^2 \alpha,$$

respectively. These unstable regions are shown in Figure 3.

The upper limit on $\Delta v/v_A$ is independent of β but increases to infinity as $\alpha \rightarrow \pi/2$, i.e. as k becomes perpendicular to Δv . Therefore no matter how fast the gas moves there will always be some modes which are unstable, only the angular distribution of possible wave vectors is reduced. Since for all these modes m is purely imaginary (to lowest order in $1/a$) the perturbations do not propagate with respect to the higher density medium 1.

3. Consequences

It is fairly probable that because of the Kelvin-Helmholtz instability a boundary layer of the disk will become turbulent. Thus we expect turbulent

diffusion to occur over most of the disk's surface. Wherever the instability is at work it mixes disk matter into the magnetic field.

Within the region $\Delta v \leq c_s$, i.e. close to the radius of corotation r_c , the neutron star's dipole field can without any large deformation take up the angular momentum of the matter diffusing onto the field lines. On the other hand mixing of field lines and matter outside the region $\Delta v \leq c_s$, would strongly wind up and distort the field, because there

$$\varrho_1 (\Delta v)^2 \gg \varrho_1 c_s^2 \cong B^2/8\pi.$$

It seems unavoidable that a large azimuthal field B_ϕ will be built up. But B_ϕ is limited by the pressure equilibrium with the unperturbed dipole field, and thus the wound-up field will expand away from the disk and decrease in strength to the dipole value. When the gas density becomes low so that the field can transfer enough angular momentum, matter will be guided to flow radially inward or outward along the field lines. The system would then stay at a state between stability and instability with velocity and density gradients in the vertical direction such that the net diffusion flow can be transported away by the dipole field above the diffusion layer.

The theoretical considerations presented above of the interaction between disk matter and the neutron star's magnetic field have some immediate consequences: Matter which gets onto the magnetic field lines exchanges angular momentum with the neutron star. A detailed quantitative model — which does not exist yet — would determine the amount δJ of angular momentum transferred to the star. Now, if the neutron star reacted as a rigid body, its own angular momentum would change corresponding to

$$I \left(\frac{2\pi}{P} \right) \dot{P}/P = \delta J,$$

where I is the moment of inertia of the neutron star, P its period, \dot{P} the time derivative of the period.

Although the whole disk surface is unstable we want to consider here the mass flow out of the disk only in a thin ring around the corotation radius r_c . This can be a reasonable description, if the mass flow falls off sufficiently rapidly with increasing r and if the inner edge of the disk is close to r_c , such

that only the region around r_c is important for the rotational state of the neutron star. For $r > r_c$ corotating matter experiences an effective outward acceleration $g_{\text{eff}} = (3GM/r_c^2)(r - r_c)$, and at a certain radius $r_{\text{eq}} \equiv r_c x_{\text{eq}}$ it leaves the magnetic field lines. The braking torque depends on this radius r_{eq} , and the angular momentum exchange is

$$\delta J_b = \dot{M}_{\text{out}} \Omega r_c^2 (x_{\text{eq}}^2 - 1).$$

The acceleration of the star is due to infalling matter for $r < r_c$

$$\delta J_{\text{acc}} = \dot{M}_{\text{in}} \Omega r_c^2.$$

If we take $\dot{M}_{\text{in}} = \dot{M}_{\text{out}} = \dot{M}$, then the rate of angular momentum change can be written as

$$\delta J = \dot{M} \Omega r_c^2 (2 - x_{\text{eq}}^2).$$

A complete balance — $\delta J = 0$ — can be achieved by setting $x_{\text{eq}} = 1.4$. In a detailed model (which we

do not have yet) x_{eq} would be determined by the balance of the magnetic field energy density and the kinetic energy density of the outflowing matter. The present status of our model leaves \dot{M}_{out} and x_{eq} essentially as free parameters.

We have shown in [6] that subsonic massflow out of the disk of a typical magnitude $\dot{M} = 10^{17}$ g/sec, in a thin ring around r_c which does not extend further than $r_c \cdot x_{\text{eq}}$, does not lead to unreasonable requirements on the physics of this turbulent diffusion process. If we take $\dot{M}_{\text{out}} = \dot{M}$, and set $\alpha = 2 - x_{\text{eq}}^2$, then the observations give spin-up time scales which are consistent with values of α around 0.05, if we take typical values for the neutron star of $M = 1 M_\odot$, $I = 10^{45}$ gcm² assuming $\dot{M} = 10 L_x/c^2$. Such a low value of α indicates that the loss and the gain of angular momentum are well balanced in most sources.

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